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Reserve Estimation Using Decline Curve Analysis for Boundary-Dominated Flow Dry Gas Wells

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Abstract
Diverse techniques have been developed to improve the estimated reserves for boundary-dominated flow dry gas wells. The various methods developed and published in various journals on how to estimate reserves range from material balance techniques to decline curve analysis. Among the various techniques, decline curves are found quite accurate in predicting good gas performance in the absence of well-known reservoir parameters. There are basically two issues that practically arise in applying decline curve analysis, particularly in boundary-dominated flow dry gas wells. First, it has been noted that it is difficult to match a decline exponent, especially at an early stage of well depletion, even with worthy quality data. Secondly, decline exponent is not constant from observation, but changes with declining production. So, the study has provided a new method based on numerical curve fitting to accurately match the Arps' decline curve function, even at the early depletion stage, and account for the changing decline exponent. Once the match objective is satisfied, future predictions can be made with a reasonable degree of assurance. Finally, the study showed that for the Arps' decline equation to be valid, the decline exponent must be between 0 and 1.

Keywords Decline curve analysis · Dry gas well · Boundary-dominated flow · Numerical curve fitting

1 Introduction

As part of the gas reservoir development plan, the utmost imperative requirements are the estimation of reserves and prediction of gas sources or reservoirs of the entire or certain fields. Various techniques have been developed lately to try to improve estimation of reserves [1]. Usually, most information about the history of production is available to start analyses on reservoir reserves using decline curve analysis [2,3].

The different methods developed and published in a variety of journals for the evaluation of reserves range from material balance approach to decline curve techniques analysis. Among the techniques, gas well performance can accurately be forecasted using decline curve techniques in absence of some reservoir parameters. It seems to be a vital method for making future forecasts and assessing the original gas in place and hydrocarbons reserves [4]. These estimates are necessary for volumetric reservoirs in determining if a particular project is economically and financially viable [5,6]. According to Ling and He [7], Arps explains that the decline curve analysis is a prediction method which extrapolates the production trends. They presented empirical conditions and relationships to evaluate rate behaviour over time.

In the early 1980s, Fetkovich introduced an analysis of decline curve using type curves. This, in fact, is a graphical method for visual matching or comparison of observational data using pre-plotted curves on a log–log diagram [8]. The type curve was applied to both transient flow period and boundary-dominated flow period. The type-curve analysis was designed for rapid performance evaluation when wells were produced under a constant bottom-hole pressure and could be used to predict future performance forecast.

In 1985, Carter introduced a type curve which featured Fetkovich's type specially designed for gas reservoirs. The technique was based on numerical solution of gas depletion in finite reservoir for radial symmetrical linear flow settings at constant bottom-hole pressures [7]. Carter introduced a

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lambda sign that reflects the size of the pressure reduction/drawdown. Cater type curve was not as direct and all-purpose when compared with how Fetkovich provides understanding and implicit guidance for analysing reservoir data. Carter also introduced other type curves that could handle the changes often associated with gas properties corresponding to pressure [9].

A few years later, Fraim and Wattenbarger in Chapra and Canale [10] studied gas wells and recognized a standard pseudo-time function to force the data from the reservoir gas to match the liquid solution for a stable well-flow pressure introduced by Fetkovich. However, this calculation involves iteration which is time-consuming [11]. Palacio and Blasingame [12] raised the question of variable non-constant wellbore pressure in boundary-dominated flow gas wells [13].

1.1 Current Challenges

This study is informed by the several unaddressed limitations in reserve estimation from decline curve analysis of boundary-dominated flow wells and multilayered gas well. One of the limitations among others is:

1. The difficulty of finding a proper nonlinear algorithm to tune the Arp's decline equation to match historical gas production. For this reason, most applications opt for type-curve matching whose inefficiency to adequately predict tight gas behaviour has been widely criticized.

The current practice in the industry provides the user with three types of decline curves to choose the one to fit. The question is, "how does the user know arbitrarily the type of decline depletion the reservoir is undergoing?" Further, it must be understood that the curvature of a decline could change during production. Accordingly, the study not being oblivious that the exponential and harmonic declines are limiting subsets of the hyperbolic decline opt to rigorously fit the hyperbolic decline. If the decline exponents are zero or very close to zero, then the decline is exponential. The decline is harmonic if the decline exponent is 1 or reasonably close to 1. This allows the depletion history tune itself adaptively to the most suitable decline curvature.

In the view of Poston and Poe [14], "decline curve analysis is probably the most widely used and least understood production forecasting technique currently in use in the industry. Analysis of decline curves provides an unsophisticated reservoir evaluation tool, but profound economic decisions often rest on the outcome of the prediction". The reason is not farfetched. The basic concept behind decline curve analysis is nonlinear curve fitting. And the mathematics of nonlinear curve fitting has generated all kinds of reconciled conflict as to the technique of evaluating the parameters of the nonlinear function. This study therefore resolves this conflict by describing an automatic self-adaptive computer nonlinear regression algorithm. It is based on nonlinear multi-parameter iteration method that discovers the minimum of a function that is expressed as the sum of the squares of the nonlinear real-valued function.

This study provides a new method to accurately match the Arp's decline curve function, even at the early depletion stage. Once the match objective is satisfied, future predictions can be made with a reasonable degree of assurance.

2 Empirical Review

2.1 Decline Curve Analysis

Declining production is caused by changes in bottom-hole pressure, productivity index, the efficiency of vertical, and horizontal fluid lift equipment, among others [15]. Declining production with respect to decreasing rate is expressed by:

$$D = -\frac{1}{q}\frac{\mathrm{d}q}{\mathrm{d}t} = -\frac{\mathrm{d}\ln q}{\mathrm{d}t} \tag{1}$$

where

q = flow rate
D = rate of nominal decline

Cutler argues that most of the decline curves usually encountered are hyperbolic, and the values of b vary from 0 to 0.7. Arps reduced the maximum value of this interval to 0.4. Lefkovit and Mathews found that for some conditions of gravity drainage $b = 0.5$. Fetkovich (1973) proved that the analysis of the hyperbolic decline curve has a theoretical basis. It also uses hyperbolic decline as a diagnostic method to show that the fall index ranges from 0 to 0.5 for gas reservoirs and 0.3333 to 0.6667 for reservoirs driven by gas.

There are two ways of predicting the decline exponent: the Arp decline curve analysis and the type-curve decline curve analysis. The Arp's method varies decline exponent and decline rate in the Arp's equation to achieve a match between production history and the Arp's equation. This type of history matching is daunting because the Arp's equation is an ill-posed nonlinear equation.

2.2 Arp's Decline Curve Analysis

In 1945, Arps working with Eq. 2 anticipated that the curvature in the rate-time decline curve can be articulated mathematically with the hyperbolic equation defined below

$$q = \frac{q_i}{(1 + bD_i t)^{1/b}} \tag{2}$$

q_i is the initial flow rate.

Though the hyperbolic decline is most common, yet the exponential and harmonic declines are applied in practice due to their simplicity.

It is the intent of this study to effectively match the past trend with the Arp's equation. Against this limitation, care must be taken to ensure that the technique is not improperly applied, as has been observed in different literatures. To remark, the conditions to ensure that the past production trends remain unchanged for the application of decline curve analysis are: the late time, constant bottom-hole pressure, and at or near capacity conditions. The late-time condition defines the pseudo-steady-state condition in which the well is in a boundary-dominated condition; that is, the well is draining a constant drainage area.

2.3 Type-Curve Decline Analysis

This analysis is based on the fact that a graph of $\log \Delta P$ versus $\log t$ has the same shape as the graph of $\log P_D$ versus $\log t_D$, but are parallel to the defined or given shift on the two axes.

Fetkovich projected that the dimensionless-variables approach can be stretched for use in decline curve analysis to streamline the calculations [8]. The variables for decline curve dimensionless flow rate, $q_{Dd} = q/q_i$ was introduced and decline curve dimensionless time, $t_{Dd} = D_i t$. Arps' relationships, during the steady-state or semi-steady-state flowing periods.

In furtherance, Fetkovich imposed the transient constant terminal pressure solution to the dimensionless form of the diffusivity equation and presented the dimensionless decline rate at different dimensionless $r_D = r_e/r_{wa}$ and the dimensionless decline time thus:

$$q_{Dd} = \frac{qi}{\frac{kh(P_i - P)}{141.2B\mu\left[\ln r_D - \frac{1}{2}\right]}} = q_D\left[\ln r_D - \frac{1}{2}\right] \tag{3}$$

Fetkovich arrived at his unified type curve by combining the transient analytical solution and the empirical pseudo-steady-state solution.

Compilation of the late-time data gives an indication of the reserves, which are a direct function of the drainage radius r_e. Knowing the coincidence of the drainage radius and the transient data match, we can calculate the effective radius of the well using the r_e/r_w parameter, and from there we can obtain the skin factor S using $r_{wa} = r_w e^{-S}$.

Initially, Fetkovich developed type curves for gas and oil wells producing at constant pressure. Carter [16] published a new set of type curves developed for specialized analysis of gas well data [17]. Carter noted that the pressure change due to the fluid properties and nature have a great influence in predicting the gas reservoir productivity and performance. This innovation is a change in the gas viscosity–compressibility product $\mu_g c_g$, which Fetkovich neglected. Carter recognized yet another set of decline curves for a boundary-dominated flow that practises a new correlating parameter λ to characterize the changes in $\mu_g c_g$ during depletion. The λ parameter, called the dimensionless drawdown correlating parameter, is selected to reflect the measure of pressure drawdown on $\mu_g c_g$ and represented as follows:

$$\lambda = \frac{\left(\mu_g c_g\right)_i}{\left(\mu_g c_g\right)_{avg}} = \frac{\left(\mu_g c_g\right)_i}{2}\left[\frac{m\left(P_i\right) - m\left(P_{wf}\right)}{\frac{P_i}{Z_i} - \frac{P_{wf}}{Z_{wf}}}\right] \tag{4}$$

For $\lambda = 1$, it shows a negligible drawdown effect and corresponds to the $b = 0$ on the Fetkovich exponential decline curve. Values of λ are the range between 0.55 and 1.0.

Fetkovich type curve does not take into account variations of bottom-hole flowing pressure for a transient regime; this limitation is considered only on cases of boundary-dominated flow and it is accounted for empirically. Moreover, for the gas wells, varying PVT properties and the reservoir pressure were not considered. In 1993, Palacio and Blasingame developed a novel method for changing production data from gas wells, bottom-hole flowing pressure, and variable rates into equivalent constant rate liquid data that can allow liquid solutions to be applied when modelling a gas flow.

This method applies a form of material balance time which requires only the harmonic decline for matching type curve.

3 Model Development

3.1 Methodology

Gas Decline Exponent Estimation before Decline Curve Analysis

This study performs its analysis by numerically fitting the Arps' decline function to obtained gas decline exponent, decline rate, and initial gas flow rate from which initial gas in place is estimated. Being that a numerical approach is desired, an estimate of decline exponent is required to initialize the tuning algorithm. The values of decline exponent and decline rate lie between 0 and 1. So, it would not be out of place to initialize decline exponent and decline rate as 0.5. But such initialization may lead the algorithm to converge to the nearest local minimum, which may not be the required minimum. To illustrate, suppose for the production trend, the Arps' decline curve has local minima at decline exponents of 0.6 and 0.8, then an initial guess of 0.5 will converge the algorithm to 0.6 in lieu of 0.8.

Table 1 OKOH PVT definition

Input	
Condensate to Gas Ratio	0
Condensate Gravity (deg API)	35
Gas Gravity (sp gr)	0.7
Temperature (deg F)	200
Mole H2S (percent)	0
Mole CO2(percent)	0
Mole N2 (percent)	0
Water Salinity (ppm)	29028

Calculate PVT

Output					
time [days]	Pressure [psia]	Z Factor	Gas FVF [cuft/Scf]	Gas Viscosity [cp]	Gas Density [lb/cuft]
0	5000	0.9934	0.00371	0.028018	14.4077
0.008191	4805.56	0.9778	0.0038	0.027372	14.0686
0.016383	4786.07	0.9762	0.00381	0.027307	14.0337
0.032767	4764.51	0.9745	0.00382	0.027235	13.9948
0.065535	4741.96	0.9728	0.00383	0.027159	13.954
0.131071	4718.93	0.971	0.00384	0.027081	13.912
0.262143	4695.63	0.9692	0.00385	0.027002	13.8692
0.524287	4672.19	0.9673	0.00386	0.026922	13.8259
1.04858	4648.6	0.9655	0.00388	0.026842	13.7821
2.09715	4624.81	0.9637	0.00389	0.026761	13.7376
4.1943	4600.83	0.9619	0.0039	0.026679	13.6924
8.38861	4576.98	0.96	0.00391	0.026597	13.6473
16.7772	4553.54	0.9583	0.00393	0.026517	13.6026
33.5544	4529.99	0.9565	0.00394	0.026436	13.5574
67.1089	4502.94	0.9544	0.00395	0.026343	13.5051
134.218	4462.04	0.9514	0.00398	0.026201	13.4254
268.435	4386.81	0.9459	0.00402	0.025939	13.2763

Table 2 MBAL PVT definition

PVT Calculations

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Temperature	Pressure	Z Factor	Gas FVF	Gas Viscosity	Gas Density
deg F	psia		ft3/scf	centipoise	lb/ft3
200	4386.81	0.945176	0.00402247	0.0249736	13.2956
200	4462.04	0.950373	0.00397639	0.0252249	13.4497
200	4502.94	0.953236	0.00395214	0.025361	13.5322
200	4529.99	0.955143	0.0039364	0.0254508	13.5863
200	4553.54	0.956812	0.00392289	0.0255288	13.6331
200	4576.98	0.958482	0.00390961	0.0256064	13.6794
200	4600.83	0.960188	0.00389627	0.0256852	13.7263
200	4624.81	0.961912	0.00388302	0.0257642	13.7731
200	4648.6	0.963631	0.00387005	0.0258425	13.8193
200	4672.19	0.965342	0.00385735	0.02592	13.8648
200	4695.63	0.96705	0.00384489	0.0259969	13.9097
200	4718.93	0.968755	0.00383265	0.0260732	13.9541
200	4741.96	0.970447	0.0038207	0.0261485	13.9978
200	4764.51	0.972111	0.00380913	0.026222	14.0403
200	4786.07	0.973708	0.0037982	0.0262922	14.0807
200	4805.56	0.975156	0.00378842	0.0263556	14.117
200	5000	0.989852	0.00369597	0.0269826	14.4702

Table 3 OKOH rate-time history match

Time (days)	Actual rate (MScf/day)	Model rate (MScf/day)	D	D^2	Time (days)	Actual rate (MScf/day)	Model rate (MScf/day)	D	D^2
0	0.5412	0.5412	0	0	0	0	0	0	0
0.008191	0.5421	0.5412	0.0009	8.1E−07	1	18323.393	18323.3923	0.0007	4.9E−07
0.016383	0.542099	0.5412	0.000899	8.08201E−07	4	18321.367	18321.36619	0.00081112	6.57916E−07
0.032767	0.542102	0.541198798	0.000903202	8.15773E−07	10	18324.979	18324.97811	0.000893202	7.97809E−07
0.065535	0.542099	0.541197597	0.000901403	8.12528E−07	28	19360.236	19360.2351	0.000900003	8.10006E−07
0.131071	0.542097	0.541195193	0.000901807	8.13256E−07	50	19184.846	19184.8451	0.000900807	8.11453E−07
0.262143	0.542	0.541190386	0.000809614	6.55474E−07	75	20532.35	20532.3492	0.000799614	6.39382E−07
0.524287	0.542	0.541180773	0.000819227	6.71133E−07	100	20436.625	20436.62414	0.000855227	7.31413E−07
1.04858	0.541996	0.541162748	0.000833252	6.94308E−07	150	15258.49	15258.4892	0.000803252	6.45213E−07
2.09715	0.541902	0.541125499	0.000776501	6.02953E−07	200	18077.467	18077.46624	0.000755501	5.70781E−07
4.1943	0.541801	0.541051009	0.000749991	5.62486E−07	250	16743.32	16743.31925	0.000750001	5.62501E−07
8.38861	0.5417	0.540900858	0.000799142	6.38628E−07	300	13563.838	13563.8372	0.000800142	6.40227E−07
16.7772	0.541401	0.540603082	0.000797918	6.36673E−07					
33.5544	0.540699	0.540005623	0.000693377	4.80772E−07					
67.1089	0.539399	0.538815078	0.000583922	3.40965E−07					
134.218	0.536703	0.536440666	0.000262334	6.8819E−08					
268.435	0.531299	0.531722006	−0.000423006	1.78934E−07					

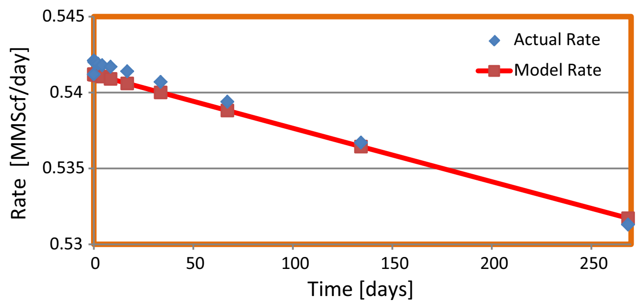


Fig. 1 OKOH History Match Plot

3.2 Development of Nonlinear Regression Algorithm

The Levenberg–Marquardt algorithm is first seen as a mixture of vanilla gradient descent and Gauss–Newton iteration [18–20]. Vanilla gradient descent is the simple and spontaneous method to find a minimum of a function. The Parameter update is accomplished by including the negative gradient at each step,

$$c_{i+1} = c_i - \nabla f \tag{5}$$

Simple gradient descent is affected by various convergence problems. This condition can be improved upon by using Newton's method:

$$c_{i+1} = c_i - H^{-1} \times \left[J^T \times D \right] \tag{6}$$

Provided that the residuals D are small, the Hessian H need not be evaluated exactly, but can be approximated essentially by the equation given as

$$H = J^T \times J \tag{7}$$

Where the Jacobian is defined as

$$J = \frac{\partial \hat{q}}{\partial c} \tag{8}$$

In fitting the production rate Arps' function $\hat{q}(t, c)$ of an independent time variable t and a vector of m regression constants c to a set of n production rate history (t, q), the objective is to find the update

$$c_{i+1} = \arg\min \{F(c)\} \tag{9}$$

$F(c)$ is defined by the Euclidian norm

$$F(c) = \frac{1}{2} \left\| \frac{f(c)}{w} \right\|^2 \tag{10}$$

where

$$f(c) \equiv D = q(t) - \hat{q}(t) \tag{11}$$

w is the weighting factor defined as

$$w = \sqrt{\frac{D^T D}{n - m + 1}} \tag{12}$$

Here, the vector of m regression constants c represents the vector elements with instantaneous decline rate and instantaneous decline exponent. In this case, the Jacobian is the $n \times 2$ matrix of the derivatives

$$\frac{\partial \hat{q}}{\partial D_i} = -\frac{q_i t}{(1 + bD_i t)^{1+1/b}} = -\frac{qt}{1 + bD_i t} \tag{13}$$

and

$$\begin{aligned} \frac{\partial \hat{q}}{\partial b} &= \frac{q_i}{b(1 + bD_i t)^{1/b}} \left[\frac{\ln(1 + bD_i t)}{b} - \frac{D_i t}{1 + bD_i t} \right] \\ &= \frac{q}{b} \left[\frac{\ln(1 + bD_i t)}{b} - \frac{D_i t}{1 + bD_i t} \right] \end{aligned} \tag{14}$$

Starting with an initial guess of the regression constants, the target is to find a perturbation

$$\text{pert} = \left[H + \lambda \cdot \text{diag}(H) \right]^{-1} \times \left[J^T \times (w \cdot D) \right] \tag{15}$$

to the regression constants that would give a new and hopefully a better match for the objective function. Provided that the residuals D are small, the Hessian H need not be evaluated exactly, but can be approximated essentially by the equation given as

$$H = J^T \times w \cdot J \tag{16}$$

With λ being the blending factor, the Levenberg–Marquardt update is

$$c_{i+1} = c_i + \text{pert} \tag{17}$$

If the error goes down following an update, λ is reduced and the algorithm degenerates to Gauss–Newton update. On the contrary, if the error amplifies following an update, λ is increased and the update becomes gradient descent iteration.

3.3 Proposed Method of Analysis

Generally, the analysis will proceed from the characterization of dry gas PVT properties. The study will utilize numerical algorithm and hence initialization of instantaneous decline

Table 4 Comparison of excel solver software & OKOH MODEL

Time (days)	Actual rate (MMSCFD)	Excel solver model rate (MMSCFD)	OKOH model rate (MMSCFD)	% Deviation between excel Solver software & Study model
0	0.5412	0.5412	0.5412	0
0.008191	0.5421	0.541199708	0.5412	−5.39542E−05
0.016383	0.542099	0.541199416	0.5412	−0.000107908
0.032767	0.542102	0.541198831	0.541198798	6.09757E−06
0.065535	0.542099	0.541197662	0.541197597	1.20104E−05
0.131071	0.542097	0.541195325	0.541195193	2.43905E-05
0.262143	0.542	0.541190649	0.541190386	4.85966E−05
0.524287	0.542	0.541181299	0.541181975	−0.000124912
1.04858	0.541996	0.541162599	0.541162748	−2.75333E−05
2.09715	0.541902	0.541125203	0.541125499	−5.47008E-05
4.1943	0.541801	0.541050423	0.54105221	−0.000330283
8.38861	0.5417	0.540900917	0.54090326	−0.000433166
16.7772	0.541401	0.54060212	0.540607883	−0.001066034
33.5544	0.540699	0.540005375	0.540015216	−0.001822389
67.1089	0.539399	0.538815279	0.538834221	−0.003515491
134.218	0.536703	0.536448583	0.536478784	−0.005629803
268.435	0.531299	0.53176856	0.531797573	−0.005455945

rate and instantaneous decline exponent follow. The instantaneous decline exponent is given by

$$b_{\mathrm{E}} = \frac{1}{2}\left[1 - \frac{p_{\mathrm{wf}}}{p_i}\right] \tag{18}$$

The initialization of instantaneous decline exponent depends on the reservoir flowing pressure and initial reservoir pressure. The Arps' decline function is regressed to obtain the values of instantaneous decline rate and instantaneous decline exponent that will provide a match of the regression model and the actual rate decline. The objective of the tuning process is to find a Levenberg–Marquadrt perturbation that would update the initial estimate of D_i and b

$$c_{i+1} = c_i + \mathrm{pert} \tag{19}$$

Here, c_{i+1} is an updated column vector of D_i and b improved from their initialized state to obtain a match.

With the values of instantaneous decline rate and instantaneous decline exponent determined, cumulative gas production is evaluated from the following equation:

$$G_{\mathrm{p}} = \int_0^{\mathrm{t}} \hat{q}\,\mathrm{d}t \tag{20}$$

The gas initially in place is computed by extrapolating P/Z plot to cut the cumulative production axis. This value is checked by estimating the gas in place from the geometry of the reservoir:

$$GIIP = \pi\left(r_{\mathrm{e}}^2 - r_{\mathrm{w}}^2\right) h\theta / B_{\mathrm{gi}} \tag{21}$$

Table 5 Cumulative gas and reserve estimate

Time (days)	Gp (MMScf)	P/Z (Psia)	Reserve (MMScf)
0	0	5033.1712	6407.69231
0.008191	0.004432967	4914.7278	6407.68787
0.016383	0.00886647	4902.5212	6407.68344
0.032767	0.017733462	4888.945	6407.67457
0.065535	0.035467389	4874.6624	6407.65684
0.131071	0.070935012	4859.9876	6407.62137
0.262143	0.14186934	4845.0491	6407.55044
0.524287	0.28373432	4829.927	6407.40857
1.04858	0.567452278	4814.6122	6407.12486
2.09715	1.134820719	4799.0692	6406.55749
4.1943	2.26932779	4783.3013	6405.42298
8.38861	4.537406843	4767.5174	6403.1549
16.7772	9.06978988	4751.9055	6398.62252
33.5544	18.11955636	4736.1202	6389.57275
67.1089	36.15930069	4717.8638	6371.53301
134.218	72.00105593	4690.0029	6335.69125
268.435	142.7452933	4637.9344	6264.94701

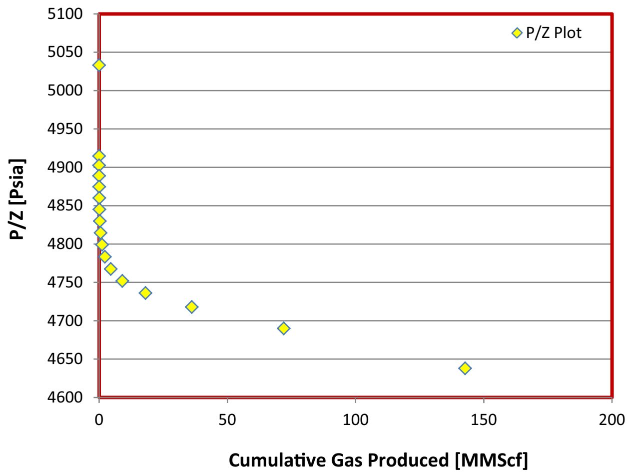


Fig. 2 OKOH's Model P/Z Plot

From here, the reserve is computed with

$$G_a = GIIP - G_p \tag{22}$$

Assumptions

1. The study is restricted to tight gas reservoir (dry gas).
2. The study did not consider the drive mechanism of the reservoir.
3. The study assumed that the gas reservoir has inherent energy, as such, the gas reservoir may not intensely require any external drive.

4 Results

4.1 Results and Discussion

Gas PVT Definition

The inputs to gas PVT characterization are shown in Table 1 using field data in Niger-Delta region.

The gas system is dry because condensate to gas ratio is zero at reservoir temperature and pressure. The gas does not contain an impurity. The study is henceforth referred to as OKOH.

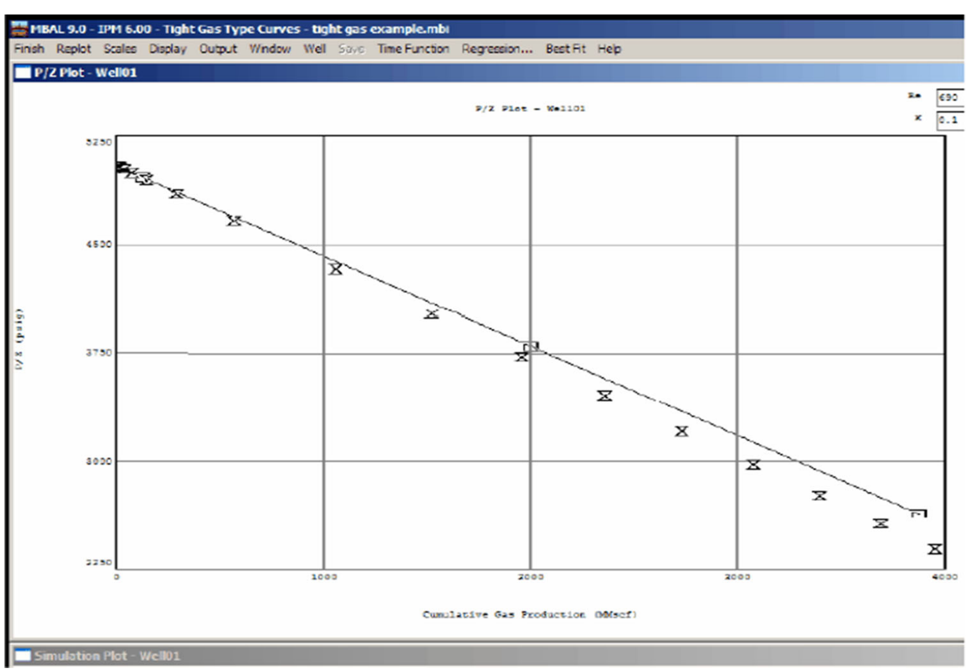
The PVT definition of the study (OKOH) is shown to closely approximate the PVT definition as computed from Petroleum Expert MBAL software (Table 2), albeit, the software usually arranges pressure data in ascending order.

4.2 History Matching

There are varieties of ways of matching actual reservoir declining production rate with the Arps' decline model, some of which are matched using different types of type curve, dimensionless pressure plot, p/Z plot, and nonlinear regression analysis method. The nonlinear regression method used in this study proves to be efficiently robust and computer based. It involves tuning the Arps' equation to find the values of instantaneous decline rate and instantaneous decline exponent that would match the reservoir declining production rate.

In order to closely initialize the regression constants, the equivalent instantaneous decline exponent is computed from

Fig. 3 MBAL's P/Z Plot



$p_i = 5000$ Psia, $p_{\mathrm{wf}} = 4386.81$ Psia as $b = 0.06132$. With the initialization, a history match was performed as shown in Table 3 and Fig. 1.

The tuning resulted in $D_i = 6.5816 \times 10^{-5}$. Indeed, the algorithm of this OKOH model proves to match the tuning capabilities of this commercial software. The excel solver's nonlinear regression algorithm result for history matching is tabulated in Table 3; and Table 4 shows the comparison of the history marching. The percentage deviation between these two algorithms is -0.0035 to 0.0000486%.

A close look at Fig. 1 reveals an important property of tight gas reservoir; that is, a small rate change. Thus, the tight gas reservoir has low permeability resulting in uneconomical flow rates, especially at the early time. The reservoir used for this study has a permeability of 0.101654mD. It is this small rate change with increasing production time that makes tuning difficult. But, as has been shown, the study's tuning algorithm matched with standard error as low as 7.74×10^{-4}.

4.3 Reserve Estimate

The cumulative gas produced at the stock tank is evaluated, and from that the reserve is estimated as shown in Table 5.

The initial gas in place is extrapolated from the P/Z against cumulative production plotted to be 6407.69MMScf as in Fig. 2.

For the study's reservoir with a porosity of 0.1 producing from pay thickness of 200 ft, drainage radius of 641.086 ft and well radius of 0.354 ft, the initial gas in place, as a way of comparison, is computed from the drainage radius as

$$\pi\left(r_{\mathrm{e}}^2 - r_{\mathrm{w}}^2\right) h\theta / B_{\mathrm{gi}} = \pi\left(641.086^2 - 0.35^2\right) \times 200 \times 0.1/0.003707 = 6965.13$$

It follows, then, that the estimate of gas initially in place from the reservoir geometry assures, prima facie, the reasonability of the value computed from the study's powerful tuning algorithm. The OKOH's model P/Z plot shows again a deviant behaviour of the tight gas reservoir. At the early stage, the data point exhibits an infinite slope and then suddenly changes to a gentle straight line slope with increasing time. Yes, the Arps' decline curve behaves well at the late time. The P/Z plot from MBAL (Fig. 3) shows a crowd of data points at the early time.

It is obvious that if the domain of plot is reduced to 200MMScft as the study's domain, those crowded data points will match the study's plot. What MBAL achieved was to extend the domain to 4000MMScf in order to clearly picture the late-time behaviour. This has never been in contention as it has been shown that at the late time the Arps' equation applies. But the study is able to track the early time with the proposed equation

$$q = q_i e^{-6.5816 \times 10^{-5} t}$$

This equation is valid for the production well studied by this work alone. To apply it to other wells, the production data from the study well are entered into the program and a new b and D_i are generated.

5 Conclusion

The OKOH's model method of the numerical algorithm is able to:

1. Fit the hyperbolic form of the Arps' decline curve function and provide a new method to accurately match the Arps' decline curve function. Once the match objective is satisfied, future predictions can be made with a reasonable degree of assurance. This has removed the trial-and-error method of selecting the type of decline to fit during decline curve analysis.
2. Predict that at the early or transient period of the dry gas reservoir that the slope is steeper than at the latter period of the reservoir as predicted by the decline curve analysis.
3. Track the behaviour of boundary-dominated flow reservoir during the transient period.

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